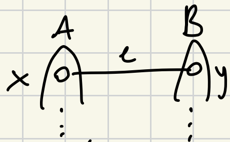
- let $G = (A \uplus B, E)$.
- $|N(X)| \geq |X|$, for all $X \subseteq A$ (1)
- G has matching M, with $|M| = |A|$ (2)
- show (1) $\Rightarrow$ (2) using induction on $a = |A|$
- $a = 1$ ✓ (I.B.)
- (1) $\Rightarrow$ (2) for some $|A| = a \geq 1$ (I.H.)
- show (1) $\Rightarrow$ (2) for $a > 1$ (I.S.)
- we know (1) holds.

we'd like to remove $v \in A$, without destroying (1)

- either $\forall X \subseteq A: \; |N(X)| > |X|$ or (C1)

  $\exists X_0 \subseteq A: \; |N(X_0)| = |X_0|$ (C2)

- (C1): choose arbitrary , remove.

  now (1) holds and (I.H.) $\Rightarrow$ exists M.

  add e to M. ✓

split graph in two smaller and apply I.H twice

- (C2): $G' = G[X_0 \uplus N(X_0)]$, $G'' = G[A \setminus X_0 \uplus B \setminus N(X_0)]$

  clearly (1) holds in G'.

  what about in G''? let $X \subseteq A \setminus X_0$, show $|N(X)| \geq |X|$.

  $|X| + |X_0| = |X \cup X_0| \overset{(1)}{\leq} |N(X \cup X_0)| = |N(X_0)| + |N(X) \setminus N(X_0)|$

  $|N(X_0)| = |X_0| \iff$ $|X| \leq |\underline{N(X)} \setminus N(X_0)| = |N(X)|$.

  $\in B \setminus N(X_0) \iff \in B \wedge \notin N(X_0)$

  use (I.H.) on G' and G''

  let $M = M' \cup M''$ in G, $|M| = |A|$. ✓