

This exercise was given to me as a practice exercise when I was taking the course Algorithms and Probability in spring semester 2023. We will solve it together in my exercise lesson on the 30th of May.

A Game of Skill

You and your friend are playing a game on a board with $n + 1$ positions, numbered from 0 to n . Each of you has a meeple, initially placed at position 0, and the goal is to move it to position n . Each of you has a six-sided die, and you take turns to roll your die and move your meeple as many positions forward as the number shown on the die. If a player is at some position x and rolls an i such that $x + i \geq n$, their meeple lands on position n and they win. You have decided to cheat and have loaded your own die, so the probability of you rolling i is p_i for every $i \in \{1, 2, \dots, 6\}$. Your friend is using a fair die, so, for them, the probability of rolling i is $\frac{1}{6}$, for every $i \in \{1, 2, \dots, 6\}$.

Your task is to answer the following questions:

1. What is the probability that after you and your friend have played one turn each, your meeples are on the same position, and that position is indexed with an even number?
2. Conditioned on your meeple landing on position 7 after your second die roll, what is the probability that your meeple landed on position 3 after your first die roll?
3. Independently of your friend's meeple's behaviour, what is the expected number of die rolls it will take you to reach position n ?

Input

The first line of the input file contains a number $t \leq 30$ of test cases. Each of the test cases is described as follows.

- It starts with a line that contains two integers n, q , separated by a space, denoting the index of the last position on the board ($2 \leq n \leq 10^3$) and the type of question you are supposed to answer ($1 \leq q \leq 3$).
- The following line contains six real numbers p_1, \dots, p_6 , separated by spaces, denoting the probability of you rolling an i with your loaded die ($0 \leq p_i \leq 1$, for all $i \in \{1, \dots, 6\}$). It is guaranteed that $\sum_{i=1}^6 p_i = 1$.

Output

For each test case, output a single line with the answer to one of the questions stated above. The q value of each test set determines which question you should output the answer to. For example, if $q = 3$, you should output a single line with the expected number of die rolls it will take you to reach position n starting from position 0. Your solution is going to be accepted if it has an absolute or relative error of at most 10^{-5} .

Points

There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 30 points, you may assume that $q = 1$ and $n \geq 10$.
2. For the second group of test sets, worth 30 points, you may assume that $q = 2$ and $n \geq 10$.
3. For the third group of test sets, worth 40 points, you may assume that $q = 3$.

Sample Case

Content of file `sample.in` :

```
3
20 1
0.3 0.2 0.1 0.2 0.2 0.0
20 2
0.3 0.2 0.1 0.2 0.2 0.0
2 3
0.5 0.2 0.2 0.1 0.0 0.0
```

Content of file `sample.out` :

```
0.0666667
0.1666667
1.5
```