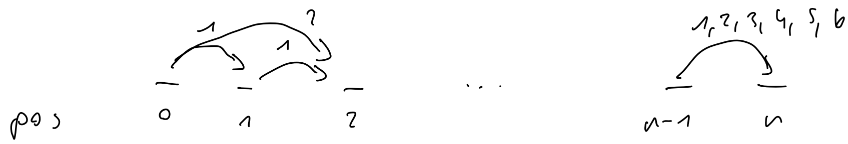
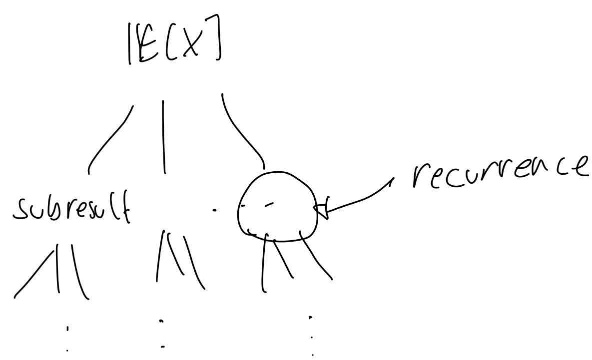
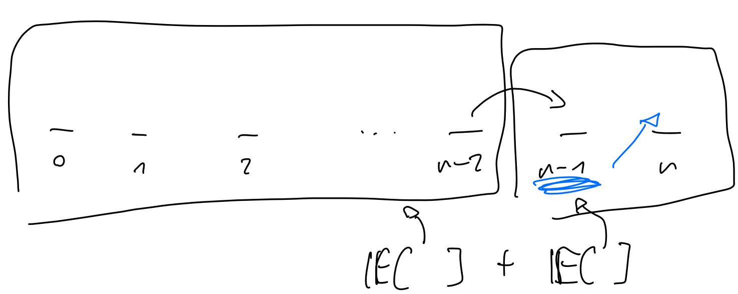
3. Independently of your friend's meeple's behaviour, what is the expected number of die rolls it will take you to reach position n?



Let X random variable that counts the # of rolls to get from 0 to n.

$\mathbb{E}[X]$?





$f(x) := \mathbb{E}[\text{"\# rolls to get from } x \text{ to } n\text{"}], \quad 0 \leq x \leq n.$

$f(n) =$ $\quad$ n to n $= 0$

/ Base case

$\underline{\quad}$
n

$f(n-1) = 1$

1, 2, 3, 4, 5, 6

1, 2

$n-1$ $\qquad$ n

$$f(n-1) = \sum_{\omega \in \Omega} X(\omega) \cdot Pr[\omega]$$

Ω is the set of "roll sequences" to get from $n-1$ to n.

$$\Omega = \{(1), (2), (3), \ldots, (6)\}$$

counts # of rolls

$$f(n-1) = \sum_{\omega \in \Omega} X(\omega) \cdot Pr[\omega]$$

$Pr[\cdot]$

$$= X((1)) \cdot Pr[(1)] \ldots$$

$\mathbb{E}[X]$

$$= 1 \cdot p_1 + 1 \cdot p_2 + 1 p_3 + \ldots + 1 \cdot p_6$$

$$= 1$$

$12 = n$

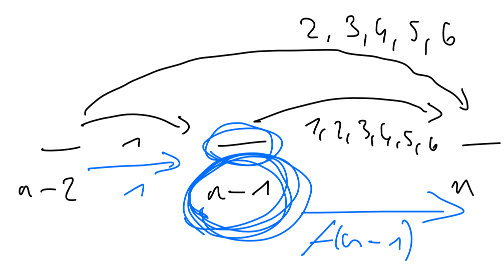
$X(6+6) = 2$

$X(1+1+\ldots+1) = 12$

$f(n) = 0$

$f(n-1) = 1$

$f(n-2) =$



$\Omega = \{(1,1), (1,2), (1,3), \ldots, (1,6)$

$(2), (3), (4), \ldots, (6)\}$

counts # of rolls

$f(n-2) = \sum_{\omega \in \Omega} X(\omega) \cdot Pr(\omega) =$

$f(n) = 0$

$= 2 \cdot p_1^2 + 2 \cdot p_1 p_2 + 2 \cdot p_1 p_3 + \ldots + 2 p_1 p_6$

$f(n-1) = \sum_{i=1}^{6} p_i = 1$

$+ 1 \cdot p_2 + 1 \cdot p_3 + \ldots + 1 \cdot p_6$

$= 2 \cdot p_1 \underbrace{(p_1 + p_2 + p_3 + \ldots + p_6)}_{\hat{=} f(n-1)}$

$+ p_2 + p_3 + \ldots + p_6$

First Idea: $f(n-2) \overset{!}{\approx} p_1 f(n-1)$ ↯

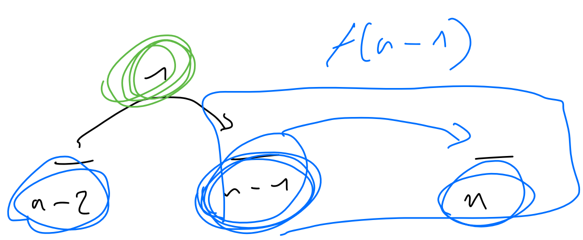
$n-2 \to n-1$

Second Idea: $f(n-2) \overset{!}{\approx} \sum_{i=1}^{6} p_i f(n-2+i)$

$$f(n-2) \overset{!}{\approx} p_1 f(n-1) + p_2 f(n) + p_3 f(n+1) + \dots + p_6 f(n+4)$$

$$= p_1 f(n-1) + p_2 f(n) + p_3 f(n) + \dots + p_6 f(n)$$

$$= p_1 \cdot 1 + p_2 \cdot 0 + \dots + 0 = p_1$$ ↯



$$p_1 (1 + f(n-1))$$

Third Idea:

$$f(n-2) = \sum_{i=1}^{6} p_i \left(1 + f(n-2+i)\right)$$

$$f(x) = \sum_{i=1}^{6} p_i \left(1 + f(\min(x+i, n))\right)$$