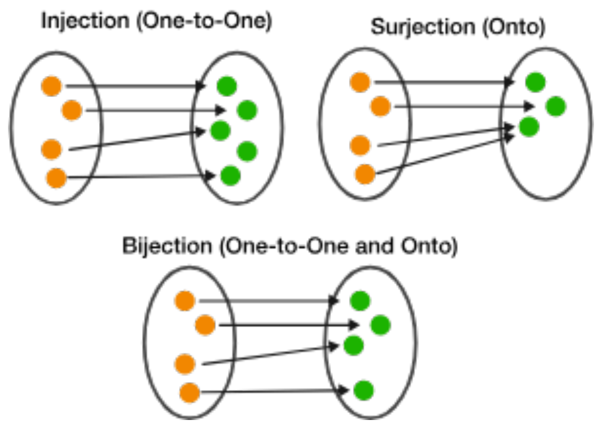
$\boxed{\text{Proof of } \mu(G/e) \geq \mu(G).}$

for all C in G/e, exists C' in G s.t.
$|C| = |C'|$.

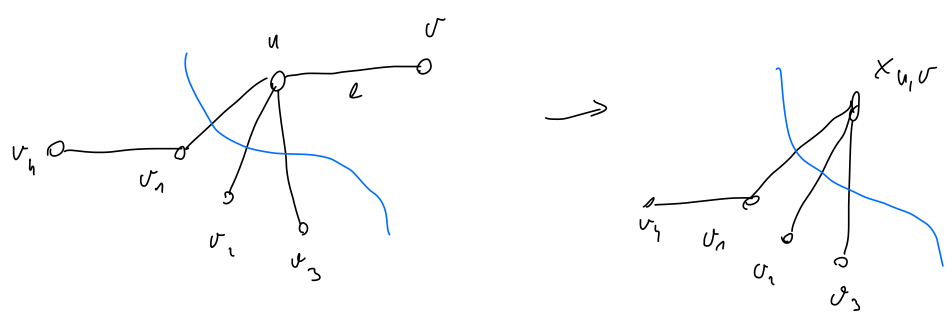


Bijection between

$\{$Kanten in G ausser denen zwischen u und $v\}$

$\longleftrightarrow$

$\{$Kanten in $G/e\}$



$\{v_1, u\} \longleftrightarrow \{v_1, x_{u,v}\}$

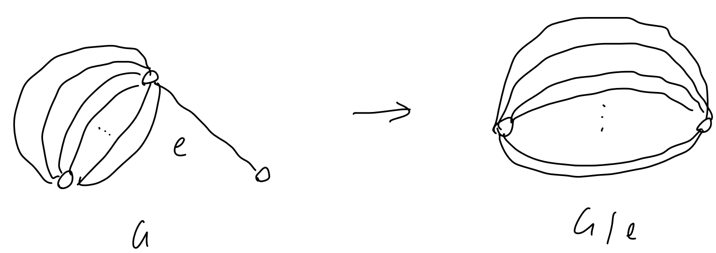
$\{v_2, u\} \longleftrightarrow \{v_1, x_{u,v}\}$

$\vdots$

$\{v_4, v_1\} \longleftrightarrow \{v_4, v_1\}$

Therefore we can't say G/e contains a smaller minimum cut C than in G, as we can always find C' in G s.t. $|C| = |C'|$.

On the other hand.



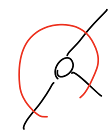
So for some e we indeed have $\mu(G) < \mu(G/e)$.

Therefore $\mu(G) \leq \mu(G/e)$.

$$\boxed{\text{Proof of } \Pr[\mu(G) = \mu(G/e)] \geq 1 - \frac{2}{n}}$$

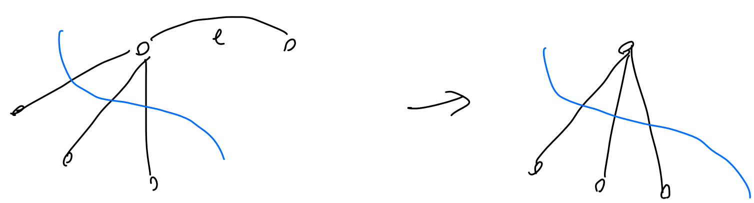
Let $k := |C| = \mu(G)$.

Note $k \geq$ minimal degree in G



$$|E| = \frac{1}{2} \sum_{v \in V} \deg(v) \geq \frac{kn}{2}.$$

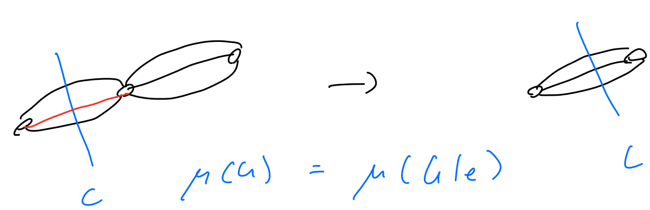
Note $e \notin C \implies \mu(G/e) = \mu(G)$

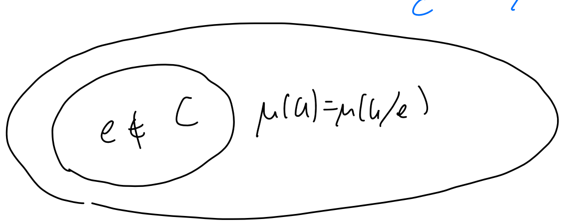


We know if $e \notin C$ then also $\mu(G/e) = \mu(G)$.

But there are also cases where $e \in C$

but still $\mu(G/e) = \mu(G)$





$\Pr[\mu(G) = \mu(G/e)] \geq \Pr[e \notin C]$.

$$\Pr[\mu(G)=\mu(G/e)] \geq \Pr[e \notin C] = 1 - \Pr[e \in C]$$

$$= 1 - \frac{|C|}{|E|}$$

$$\geq 1 - \frac{k}{kn/2} = 1 - \frac{2}{n}.$$

$\square$