Week 8 — Sheet 7

Algorithms and Data Structures

13.11.2023 — Georg Hasebe

Debriefing of Submissions

```
A(n):
    if memo[n] defined
        return memo[n]
    if n <= 4
        return n
    else
        return A(n - 1) + A(n - 3) + 2A(n - 4)
```

```
A(n):
    if memo[n] defined
        return memo[n]
    if n <= 4
        return n
    else
        memo[n] <- A(n - 1) + A(n - 3) + 2A(n - 4)
        return memo[n]
```

- Naming variable root can **decrease readability**, use current_node or curr_node or node etc. instead
- Initialization:
    - Memo <- [1…n]? ✗
    - Memo[n] <- [1…n]? ✗
    - Memo[1…n]? ✗
    - Memo[1…n] <- [-1,…,-1] etc. ✓
- Scope? Is memo[n] initialized globally? Or every recursion?

Suggestion: Read Solutions

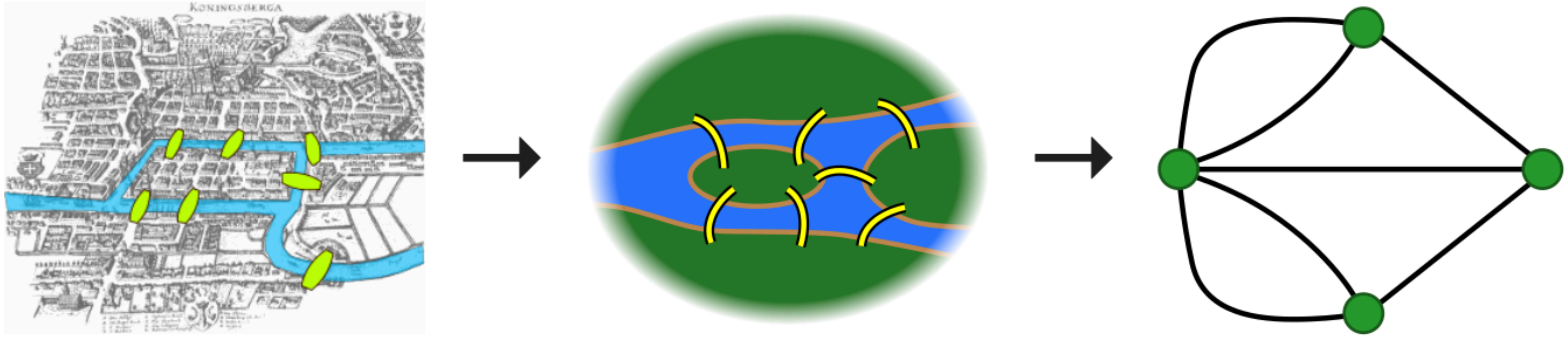
Exercise Sheet 7

Debriefing of Exercise Sheet 6

Theory Recap

Graph Theory

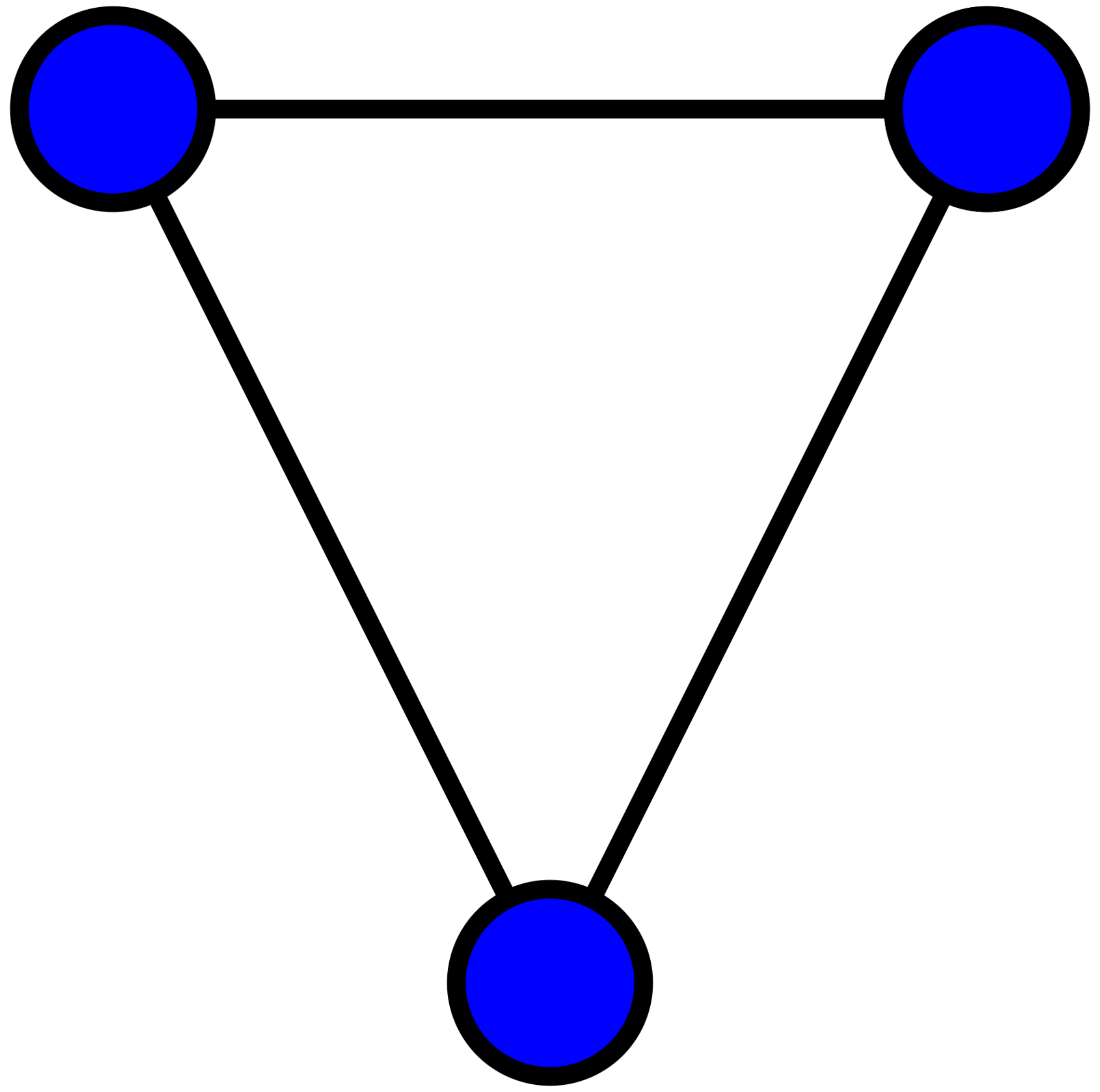
Graph Theory



[1] https://upload.wikimedia.org/wikipedia/commons/6/60/Leonhard_Euler_2.jpg
[2] https://de.wikipedia.org/wiki/Königsberger_Brückenproblem

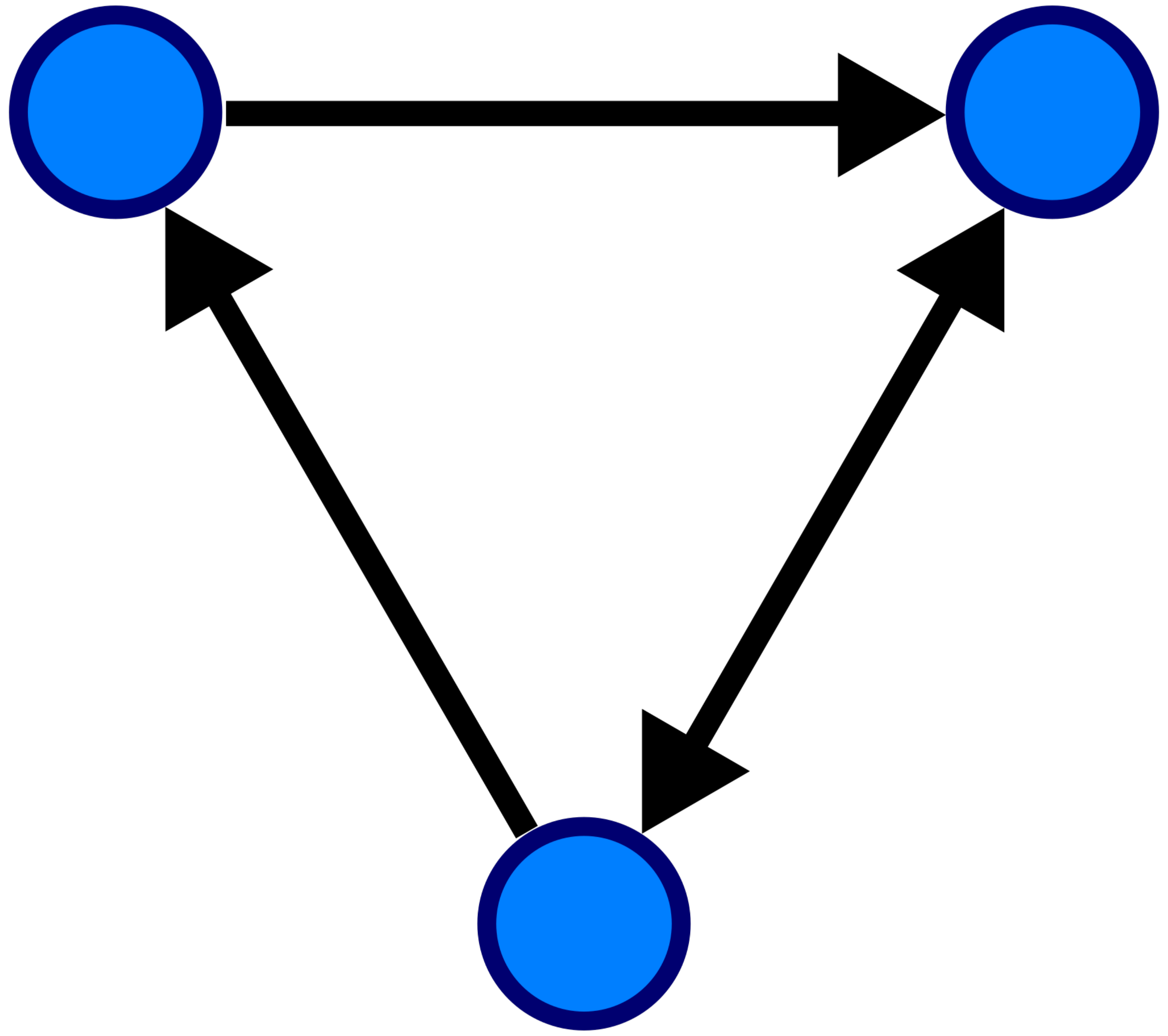
Graph Theory

- Undirected Graphs
- V a set of vertices
- $E \subseteq \{\{x, y\} \mid x, y \in V \text{ and } x \neq y\}$



https://en.wikipedia.org/wiki/Graph_theory#/media/File:Undirected.svg

- Directed Graphs
- V a set of vertices
- $E \subseteq \{(x, y) \mid (x, y) \in V^2 \text{ and } x \neq y\}$



https://en.wikipedia.org/wiki/Graph_theory#/media/File:Directed.svg

Eulerian trail (Eulerweg)

(Sometimes also Eulerian path in literature)

In graph theory, an **Eulerian trail** (or **Eulerian path**) is a trail in a finite graph that visits every edge exactly once (allowing for revisiting vertices).

A connected graph has an Euler trail. ⇔ Every vertex, except for at most 2, has even degree.

Eulerian circuit (Eulerkreis)
(Sometimes also Eulerian Cycle in literature)

An **Eulerian circuit** or **Eulerian cycle** is an Eulerian trail that starts and ends on the same vertex.

A connected graph has an Euler cycle. $\Leftrightarrow$ Every vertex has even degree.

Hierholzer's Algorithm in $O(|E|)$

Voraussetzung: Sei G=(V,E) ein zusammenhängender Graph, der nur Knoten mit geradem Grad aufweist.

1. Wähle einen beliebigen Knoten v0 des Graphen und konstruiere von v0 ausgehend einen Unterkreis K in G, der keine Kante in G zweimal durchläuft.

2. Wenn K ein Eulerkreis ist, breche ab. Andernfalls:

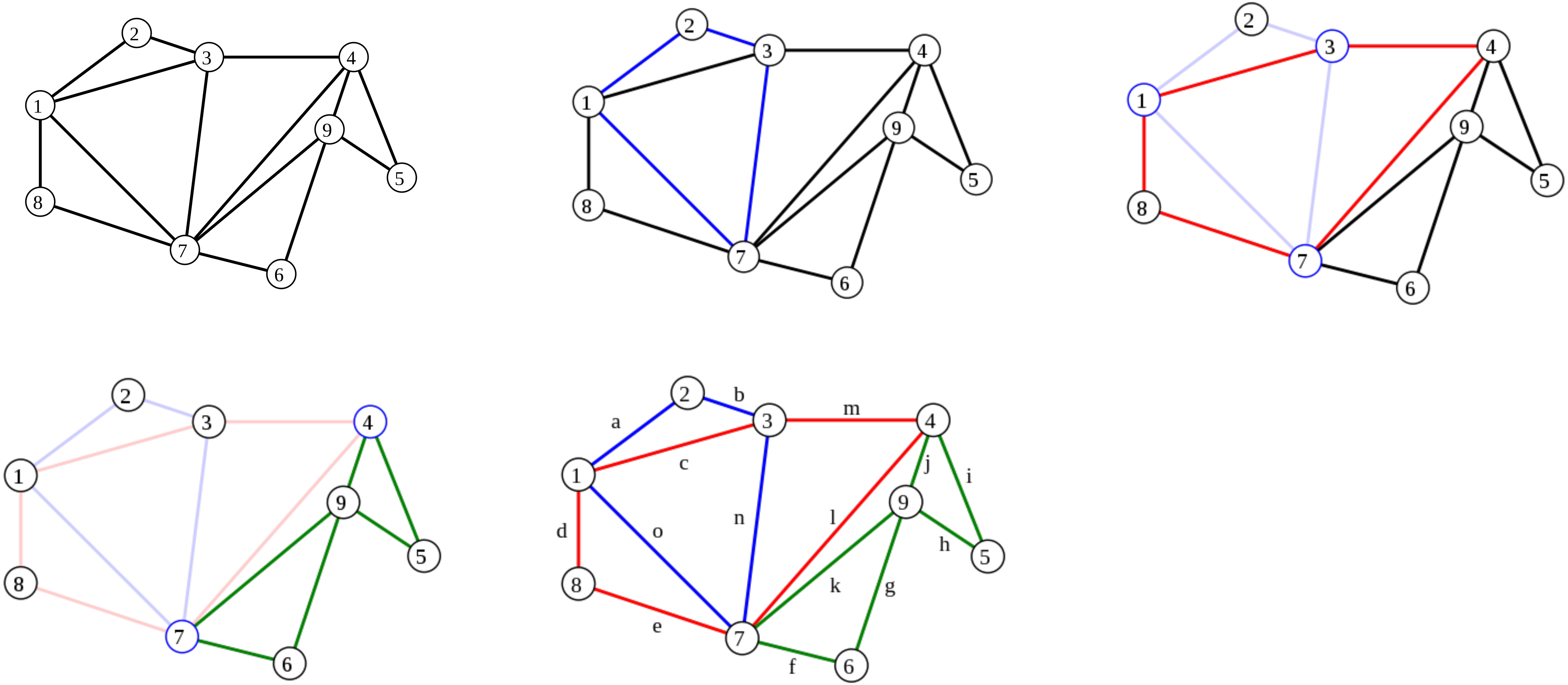
3. Vernachlässige nun alle Kanten des Unterkreises K.

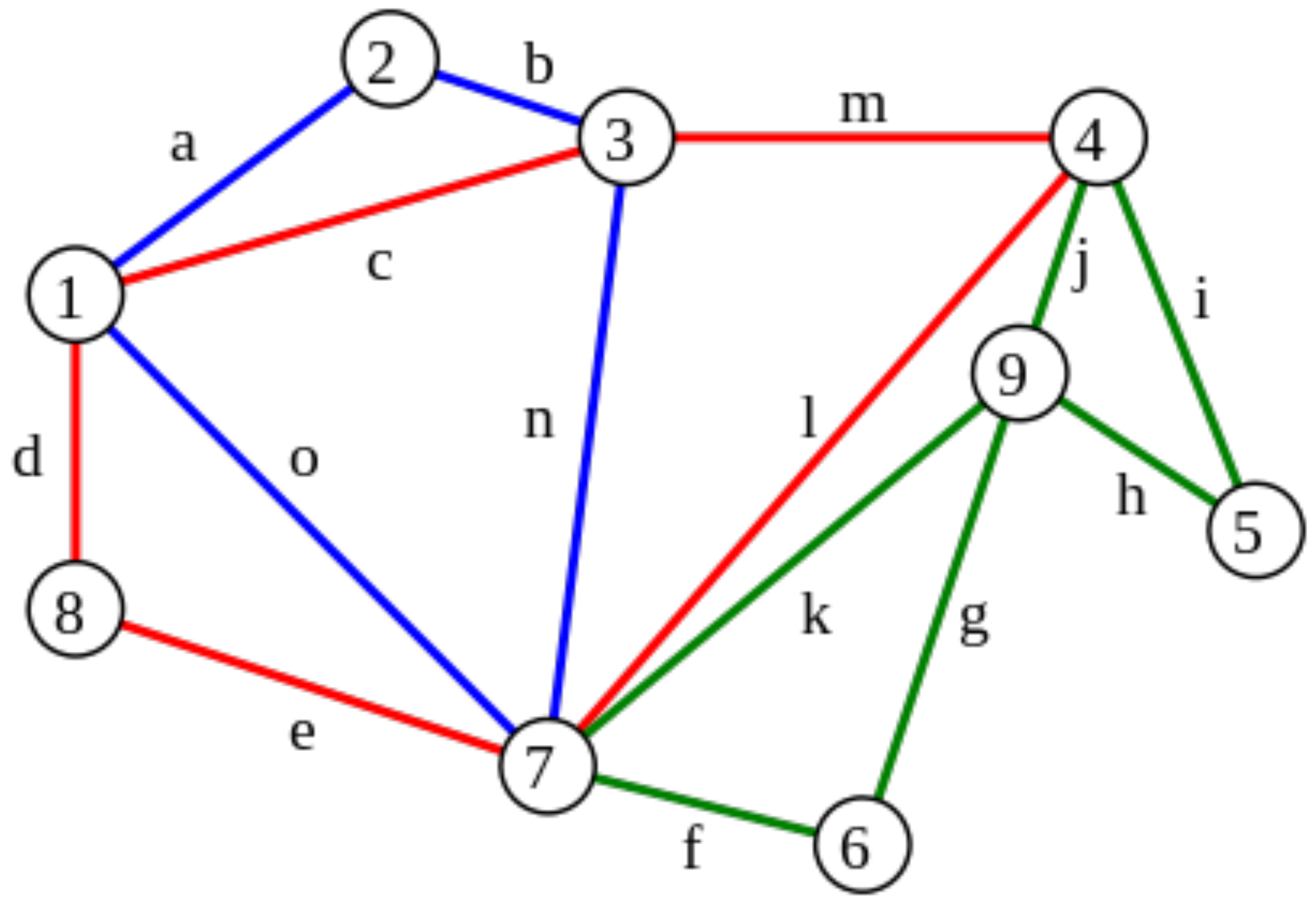
4. Am ersten Eckpunkt von K, dessen Grad größer 0 ist, lässt man nun einen weiteren Unterkreis K′ entstehen, der keine Kante in K durchläuft und keine Kante in G zweimal enthält.

5. Füge in K den zweiten Kreis K' ein, indem der Startpunkt von K′ durch alle Punkte von K′ in der richtigen Reihenfolge ersetzt wird.

6. Nenne jetzt den so erhaltenen Kreis K und fahre bei Schritt 2 fort.

Hierholzer’s Algorithm Example





(1,2,3,7,1)

↓

(1,2,3,1,8,7,4,3,7,1)

↓

(1,2,3,1,8,7,6,9,5,4,9,7,4,3,7,1)

E = (1,2,3,1,8,7,6,9,5,4,9,7,4,3,7,1)

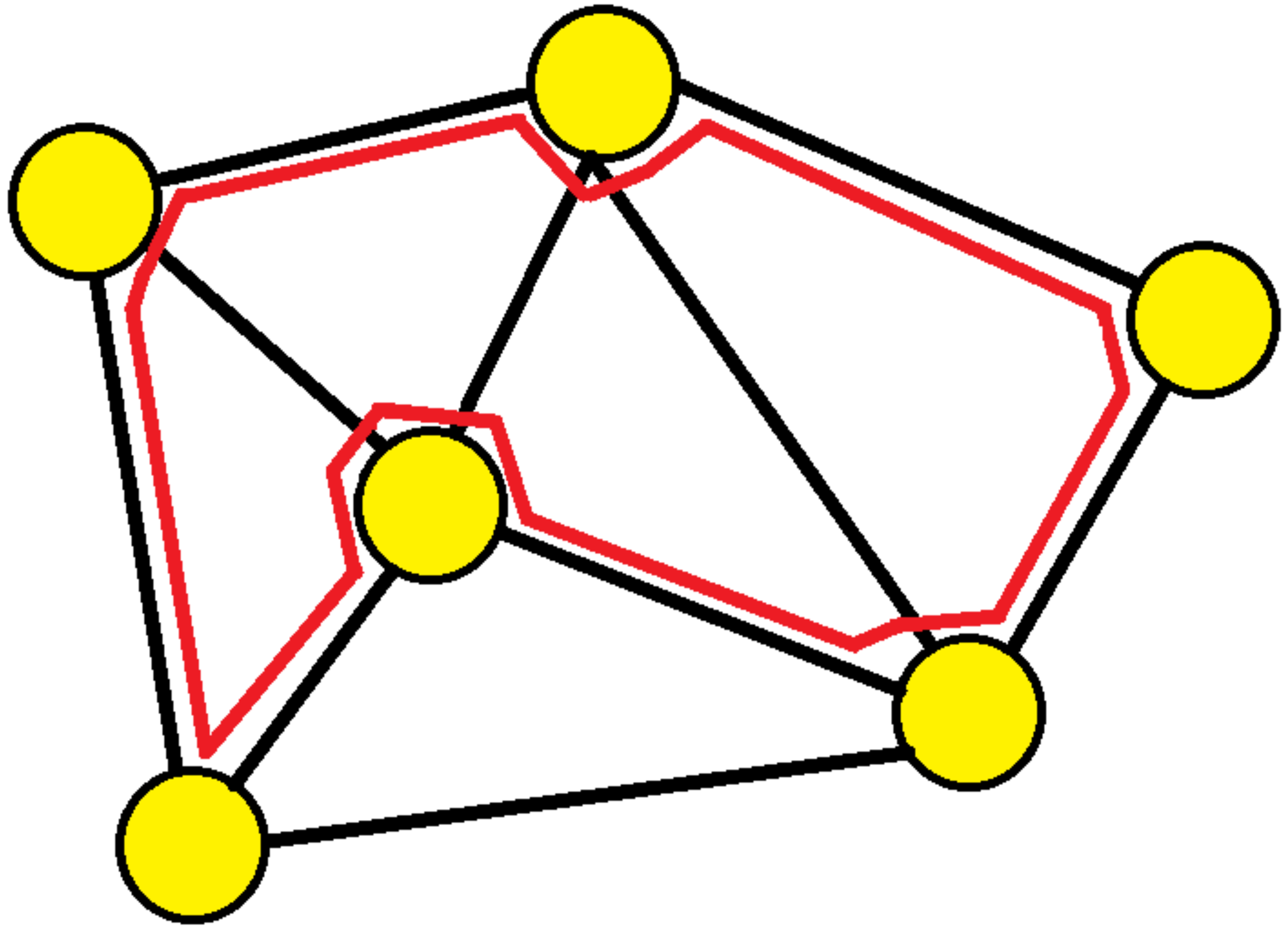
$C_{\text{blau}} = (1,2,3,7,1)$

$C_{\text{red}} = (3,1,8,7,4,3)$

$C_{\text{green}} = (7,6,9,5,4,9,7)$

Hamiltonian path and Hamiltonian cycle (curcuit)

In the mathematical field of graph theory, a **Hamiltonian path** (or **traceable path**) is a path in an undirected or directed graph that visits each vertex exactly once. A **Hamiltonian cycle** (or **Hamiltonian circuit**) is a cycle that visits each vertex exactly once.



Hamiltonian path and Hamiltonian cycle (circuit)

Determining if such paths or cycles exist is NP-Complete (very hard).

Graph Terminology

Sadly sometimes ambiguous and confusing

- Julian Steinmann’s Graph Terminology Cheatsheet
  - https://exams.vis.ethz.ch/user/jsteinmann/document/graph-terminology-cheatsheet
- Wikipedia’s Glossary of Graph Theory
  - https://en.wikipedia.org/wiki/Glossary_of_graph_theory#trail

Kahoot