(1) calculate limits ...

(2) Yes! Review definition in script.

(3) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n \leq O(n^2)$

alternative: $\sum_{i=1}^{n} i \leq \sum_{i=1}^{n} n = n^2 \leq O(n^2)$

(4) $\log(n!) = \log(1 \cdot 2 \cdot \ldots \cdot n) = \sum_{i=1}^{n} \log(i) \leq \sum_{i=1}^{n} \log(n)$

$= n \log(n)$

$\leq O(n^2 \log(n))$

(5) from (a) $\Longrightarrow$ $\log(n!) \leq n \log n$

$\Longrightarrow \lim_{n \to \infty} \frac{n^2 \log n}{n \log n} = \infty$

thus $n^2 \log n \notin O(\log(n!))$

(6) false, limit definition.

(7) script / lecture notes

(8) $\lim_{n \to \infty} \frac{2^n}{n^{2023}} = \lim_{n \to \infty} \frac{2^n}{2^{2023 \log n}} = \lim_{n \to \infty} 2^{n - 2023 \log n}$

a) $n \to \infty$ $\quad n - 2023 \log n \to \infty$, thus

$$\lim_{n \to \infty} \frac{2^n}{n^{2023}} = \infty$$

(9) script

(10) script / lecture notes

(11) True!

$$O(n) = \{ g : \mathbb{N} \to \mathbb{R}^+ \mid \exists c > 0, n_0 \in \mathbb{N} : g(n) \leq c n \quad \forall n \geq n_0 \}$$

$$O(n^2) = \{ g : \mathbb{N} \to \mathbb{R}^+ \mid \exists c > 0, n_0 \in \mathbb{N} : g(n) \leq c n^2 \quad \forall n \geq n_0 \}$$

since $\quad g(n) \leq cn \leq cn^2 \implies O(n) \subseteq O(n^2)$

(12) False!

Consider $n \in \Theta(n)$, but $n \notin \Theta(n^2)$!

Check with definitions in script!